Exploring Klotski: An Investigation of the Minimum Number of Moves in a Special Case of Slide Puzzles

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Abstract

Klotski is a special type of sliding puzzle first emerged in the early 20th century. It refers to a variety of sliding block or tile games with the goal of moving a specific block to a predetermined spot. This investigation aimed to determine the minimum number of moves by altering the game's regulation that requires a player to move a specific tile or block from one corner of the puzzle to its opposite corner. The results of the investigation proposed the use of the formula $M = 8s - 11$ in determining the minimum number of moves (M) of a square-shaped Klotski given the number of columns/rows (s) of the puzzle. Alternatively, the formula $M = 6l + 2w - 13$ was also proposed to determine the minimum number of moves (L) of a rectangular-shaped Klotski with l and w as the number of columns and rows, and $l > w$. In the case where only the product (P) and difference (d) of the dimensions are given, the formula $M_{Pd} = 4\sqrt{d^2 + 4P} + 2d - 13$ may be used to determine the minimum number of moves (M_{Pd}). It is recommended that further studies must be conducted to determine the other possible real life application of the concept and additional investigations must be done for other cases of the puzzle.

Keywords:*Arithmetic, Klotski, Minimum Number of Moves, Pattern, Sliding Puzzle*

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1 Chapter I

1.1 Introduction

Klotski is a type of a sliding puzzle that have started and gained popularity in the early 20th century. It refers to a whole group of similar sliding-block puzzles with a same objective of moving a certain block to a predetermined location.[3] Within the frame, the blocks can be moved anywhere by sliding—not turning, lifting or jumping. To complete the sliding puzzle, the starting and finishing positions are typically provided.[2]

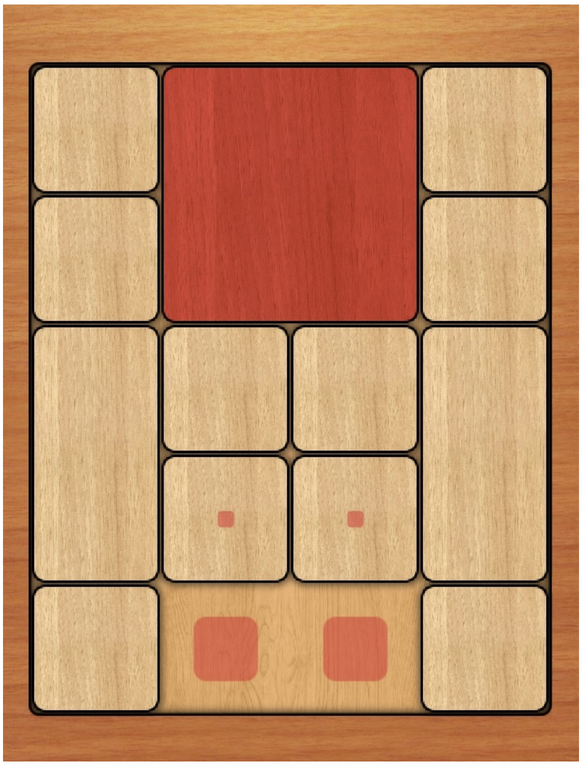

Figure 1: Sample of a Mobile Game Klotski

Upon watching a player do the Klotski puzzle by moving a tile from one place to another, the investigators observed some patterns that are related and was tackled in their Mathematics class in the previous school year. This urges them to perform a short experimentation of moving a tile in a diagonal direction in order to find the minimum number of moves and to formulate a mathematical investigation about the Klotski puzzle in hopes of discovering concepts that may contribute in the study of mathematics.

Thus, this study focuses on exploring the Klotski puzzle and to generate formulas out of the number of moves it needed to complete the puzzle. This investigation also aims to determine the minimum possible number of moves in solving a square or rectangular-shaped Klotski with the aid of the different mathematical concepts, particularly the arithmetic sequence.

Like other varieties of sliding puzzle, Klotski is also known as a challenging game that brings entertainment to people in all ages. This game also enhance an individual's problem solving skills, critical thinking skills and other mathematical related skills. Moreover, this game might not just be a "simple game" for it has the potential to unfold mathematical concepts that have a significant contribution to mathematical research.

1.2 Significance of the Study

This study adds to the corpus of information for everyone, especially for gamers, and it is noteworthy since the idea might be used in other games. Due to its usage and relation to the concept of sequence and patterns, this investigation makes fresh contribution to the field of games and geometry. If this study will be a success, not only students but everyone else may be able to benefit from this study by using the concepts in this investigation in the future and pass this knowledge to the next generation. Lastly, the investigators of this study are hoping that other investigators would find the relevance of this study into its real life application.

1.3 Statement of the Problem

The aim of the game under investigation is to slide a corner tile/block from its starting position (upper left corner) to its opposite corner (bottom right) in the least possible number of moves. In addition, each block/tile can only be moved up, down, left, right and one at a time. Diagonal movements and lifting the blocks are not permitted and the starting free space of the puzzle is located at the bottom right corner.

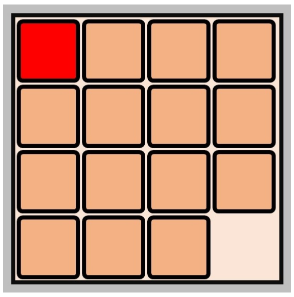

Figure 2: Sample Predetermined State of a Square Klotski

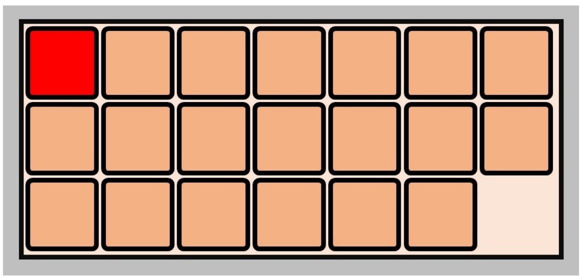

Figure 3: Sample Predetermined State of a Rectangular Klotski

Based on the aforementioned rule of the game, the following objectives were identified to facilitate the conduct of this investigation:

- To determine the minimum number of moves in the game Klotski

- To determine whether a pattern is formed by the minimum number of moves

- To create a formula in calculating the minimum number of moves for all square Klotski

- To create a formula in calculating the minimum number of moves for all rectangular Klotski

1.4 Review of Related Literature

1.4.1 Klotski and other Sliding Puzzles

A frame enclosed collection of forms makes up a sliding puzzle. The only way to move the shapes inside that frame is to slide; turning, lifting, or jumping are not permitted. The 15 puzzle, created in 1878 in the USA (by an unidentified person), was the one that actually kicked off the genre of puzzles.

Figure 4: Sample of a 3 × 3 Mobile Game Sliding Puzzle

Figure 5: Sample of a 5 × 5 Mobile Game Sliding Puzzle

The traditional Klotski puzzle is one of the notable sliding puzzle that was known as "Hua Rong Dao" in China and "Dad's Puzzler" of USA. In China, this traditional Klotski gained popularity in the 1930's based on the warrior from the Eastern Han Dynasty and well-known Chinese strategist Cao Cao. The Cao Cao piece serves as the main header in sliding block puzzles, and the other nine pieces represent armed guards and other barriers. The player's task is to overcome these challenges and deliver Cao Cao to freedom at the bottom of the board.[7] On the other hand, Dad's puzzler was invented by one of America's leading mathematician, Olive C. Hazlett during the 1920s. There is no pictures to put together or number to get in order in this puzzle, it's just simply uses pieces of wood. To play this puzzle, the large square must be moved from position A to position C in order to complete the puzzle. Also, no pieces may be turned, raised off the bottom of the frame, or jumped.[6]

The frame and base of the Klotski puzzle and Dad's Puzzler serves as the foundation upon which the puzzle is built, it also have the crucial role to constraints within which blocks can be shifted. It is commonly made out of wood or durable plastic. China's Klotski puzzle have one large block(2×2), representing the protagonist. It also have 4 medium rectangles(2×1) and 4 small squares(1×1) that can be slid in a specific direction to help the protagonist reach the exit. While Dad's puzzler's blocks have one 2×2 square, four 2×1 rectangles with the long side horizontal, two 2×1 rectangles with the long side vertical, and two 1×1 squares.[5]

The Klotski puzzle has changed over time, giving rise to a variety of versions and adaptations that continue to intrigue and test puzzle enthusiasts. Some example of these are Klotski puzzle with Animated images, Klotski puzzle with a hexagonal frame, and a Klotski puzzle that holds more than 10 blocks that the traditional one have. The well-known Klotski puzzle game was also included in the improvement of technology after becoming part of Windows 3.1's Microsoft entertainment pack and has since become a classic.[5]

A puzzle is a type of educational game that brings entertainment and challenges everyone to solve it. To find the right and enjoyable solution to a puzzle it requires the development of certain skill that may not only help solve the puzzle but also enhances one's congnitive abilities. This will put the observation skill, planning skill, critical thinking skill and patience up to a test. Playing the puzzle will help us develop our skills that we can also apply in our daily lives without taking away the fun.[5]

According to the recent article of Zhong (2023), studying new algorithms to solve sliding puzzles has become increasingly important as sliding puzzle sports have developed. Zhong along with other investigators hope to develop an additive approximation method because it is difficult to solve the puzzle optimally. The length of the answer produced by this algorithm should only be a low-order term longer than the ideal solution.[9]

1.4.2 Arithmetic Sequence

Carl Friedrich Gauss is a German Mathematician who contributed in many fields of Math and Science. When he was a young boy he was tasked to add the integers 1 to 100 by his teacher. As a Math prodigy he is, he was able to create his own formula which leads to the discovery of arithmetic sequence. It is an ordered set of numbers that have a common difference between each consecutive term.[1]

Mathematicians and others who work with numbers can solve difficult mathematical problems by using arithmetic sequences, which are employed in algebra and geometry. Arithmetic sequences can be used to solve simple or complex problems but it's necessary to have the basic understanding about it to ensure that it is applied correctly. Finding patterns, such as a specific number of extra seats each row, is how this is done.[4]

In the study of Oded Goldreich in 2011, They take into account a game in which players move various stones along an undirected graph's edges. Only one pebble may be present in each vertex at any one moment, and only one pebble may be transferred at a time (i.e., the pebble must be moved to an empty vertex). They demonstrate that it is NP-Hard to determine the shortest sequence of moves between two given "pebble configurations".[8]

1.5 Scope and Delimitations

This investigation aims to find out the minimum number of moves if a block in a Klotski puzzle is moved to a specific location from its predetermined spot. A block or tile must be diagonally moved from the upper left corner to its opposite corner (bottom right). The only legal moves are: a. sliding the block/tile up, down, left and right directions only, b. the number of blocks that can be moved is limited to one at a time. The size of the blocks are of equal measure but the dimension of the puzzle differs, which adds to the difficulty of the puzzle. Also, this study focuses on the derivation and/or creation of formulas out of the discovered pattern or sequence. Lastly, this study is not only limited to square Klotski puzzles, as it also take into consideration the rectangular Klotski puzzles with varying dimensions.

2 Chapter II

2.1 Definition

Arithmetic Sequence- a sequence where the next terms after the first term is obtained by adding a constant number. The difference between each consecutive terms is called the common difference.

Klotski- comes from the polish word "klocki" which means wooden blocks. It is a sliding puzzle that aims to move the red block to exit at the bottom of the board or to move the red block from and into a specified location.

Minimum moves- the least quantity, amount possible, assignable, allowable, or the like in moving a block from a Klotski puzzle.

Pattern- is a repeated arrangement of numbers, shapes, colors and so on. If the set of numbers are related to each other in a specific rule, then the rule or manner is a pattern.

Sliding puzzle- is a type of simple combination puzzle. It challenges the player to slide pieces along a certain routes on a board to reach a certain end arrangement.

2.2 Known Results or Conjecture

The following are the conjectures generated by the investigators in the conduct of the investigation:

• The minimum number of moves for square and rectangular with varying Klotski dimensions forms an arithmetic sequence

• The formula to find the minimum number of moves (M) of a square Klotski is $M = 8s - 11$, where s is the number of columns or rows

• The formula to find the minimum number of moves of a rectangular Klotski are:

1. $M_{Pd} = 4\sqrt{d^2 + 4P} + 2d - 13$, where d is the positive difference and P is the product between the number of rows and columns

2. $M = 6l + 2w - 13$, where l and w are the number of rows and columns, and $l > w$

3 Chapter III

3.1 Researchers' Generated Data and Results

This section presents the data that the investigators gathered, the pattern uncovered and the formulation and derivation of the formula to find the minimum number of moves on the Klotski Puzzle. The following data and results are organized and presented based on the order of the specific objectives identified in section 1.3

Data for the Minimum Number of Moves

Number of Columns

Number of Rows

X	1	2	3	4	5	6	7	8	9	10	11	12	...
1	Ø	1	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	...
2	1	5	9	15	21	27	33	39	45	51	57	63	...
3	Ø	9	13	17	23	29	35	41	47	53	59	65	...
4	Ø	15	17	21	25	31	37	43	49	55	61	67	...
5	Ø	21	23	25	29	33	39	45	51	57	63	69	...
6	Ø	27	29	31	33	37	41	47	53	59	65	71	...
7	Ø	33	35	37	39	41	45	49	55	61	67	73	...
8	Ø	39	41	43	45	47	49	53	57	63	69	75	...
9	Ø	45	47	49	51	53	55	57	61	65	71	77	...
10	Ø	51	53	55	57	59	61	63	65	69	73	79	...
11	Ø	57	59	61	63	65	67	69	71	73	77	81	...
12	Ø	63	65	67	69	71	73	75	77	79	81	85	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...

Figure 6: Minimum Number of Moves based on the Number of Rows and Columns

The minimum number of moves in a square Klotski is always obtained by traveling the main block/tile around the diagonal path of the puzzle or it follows a stair-like path. For the rectangular Klotski, a similar strategy can be used were the main block/tile should travel also around the diagonal path of the puzzle.

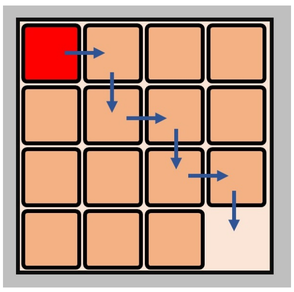

Figure 7: Path to obtain the Minimum Number of Moves of a Square Klotski

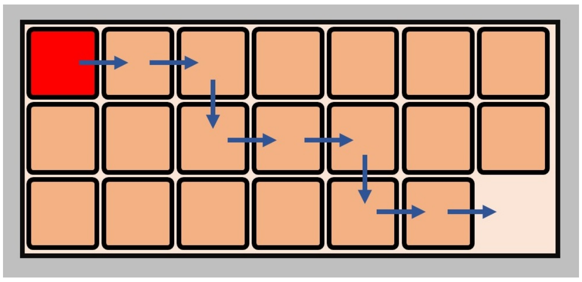

Figure 8: Path to obtain the Minimum Number of Moves of a Rectangular Klotski

Pattern for the Minimum Number of Moves

Number of Columns

Number of Rows

x	1	2	3	4	5	6	7	8	9	10	11	12	...
1	Ø	1	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	...
2	1	5	9	15	21	27	33	39	45	51	57	63	...
3	Ø	9	13	17	23	29	35	41	47	53	59	65	...
4	Ø	15	17	21	25	31	37	43	49	55	61	67	...
5	Ø	21	23	25	29	33	39	45	51	57	63	69	...
6	Ø	27	29	31	33	37	41	47	53	59	65	71	...
7	Ø	33	35	37	39	41	45	49	55	61	67	73	...
8	Ø	39	41	43	45	47	49	53	57	63	69	75	...
9	Ø	45	47	49	51	53	55	57	61	65	71	77	...
10	Ø	51	53	55	57	59	61	63	65	69	73	79	...
11	Ø	57	59	61	63	65	67	69	71	73	77	81	...
12	Ø	63	65	67	69	71	73	75	77	79	81	85	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...

Figure 9: Pattern for the Minimum Number of Moves for a Square Klotski

As shown in Figure 9, the minimum number of moves of a square Klotski (in an increasing number of sides) forms an Arithmetic Sequence whose common difference is 8.

Number of Columns

Number of Rows

x	1	2	3	4	5	6	7	8	9	10	11	12	...
1	Ø	1	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	...
2	1	5	9	15	21	27	33	39	45	51	57	63	...
3	Ø	9	13	17	23	29	35	41	47	53	59	65	...
4	Ø	15	17	21	25	31	37	43	49	55	61	67	...
5	Ø	21	23	25	29	33	39	45	51	57	63	69	...
6	Ø	27	29	31	33	37	41	47	53	59	65	71	...
7	Ø	33	35	37	39	41	45	49	55	61	67	73	...
8	Ø	39	41	43	45	47	49	53	57	63	69	75	...
9	Ø	45	47	49	51	53	55	57	61	65	71	77	...
10	Ø	51	53	55	57	59	61	63	65	69	73	79	...
11	Ø	57	59	61	63	65	67	69	71	73	77	81	...
12	Ø	63	65	67	69	71	73	75	77	79	81	85	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...

Figure 10: Pattern Symmetry

As shown in Figure 10, a symmetry of pattern was formed where the minimum number of moves of an $l \times w$ rectangular Klotski maps (and is equal to) the minimum number of moves of an $w \times l$ Klotski. This is due to the rotation and translation properties of the puzzle.

Number of Columns

Number of Rows

x	1	2	3	4	5	6	7	8	9	10	11	12	...
1	Ø	1	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	Ø	...
2	1	5	9	15	21	27	33	39	45	51	57	63	...
3	Ø	9	13	17	23	29	35	41	47	53	59	65	...
4	Ø	15	17	21	25	31	37	43	49	55	61	67	...
5	Ø	21	23	25	29	33	39	45	51	57	63	69	...
6	Ø	27	29	31	33	37	41	47	53	59	65	71	...
7	Ø	33	35	37	39	41	45	49	55	61	67	73	...
8	Ø	39	41	43	45	47	49	53	57	63	69	75	...
9	Ø	45	47	49	51	53	55	57	61	65	71	77	...
10	Ø	51	53	55	57	59	61	63	65	69	73	79	...
11	Ø	57	59	61	63	65	67	69	71	73	77	81	...
12	Ø	63	65	67	69	71	73	75	77	79	81	85	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...

Figure 11: Pattern for the Minimum Number of Moves for a Rectangular Klotski

As shown in Figure 11, the minimum number of moves of a rectangular Klotski (in an increasing number of sides) forms also an Arithmetic Sequence whose common difference is 8 just like the pattern of the square Klotski. Furthermore, the first term of each sequence in the rectangular Klotski also forms an arithmetic sequence whose common difference is 6.

Formula for the Minimum Number of Moves of a Square Klotski;

$$(n+1)(n+1) \; or \; (n+1)^2 \quad \{n | n \in \mathbb{Z}^+\}$$

Product(P)	4	9	16	25	36	49	64	81	...	$P=(n+1)^2$
Minimum No. of Moves (M)	5	13	21	29	37	45	53	61	...	$M=8n-3$

Formulation of $M = 8n - 3$

$$M = 5 + (n-1)8$$
$$M = 5 + 8n - 8$$
$$M = 8n - 3$$

Derivation of n

$$P = (n+1)^2$$
$$\sqrt{P} = \sqrt{(n+1)^2}$$
$$\sqrt{P} = n+1$$
$$\sqrt{P} - 1 = n$$
$$n = \sqrt{P} - 1$$

Minimum number of moves in terms of the product (M_P)

$$M = 8n - 3$$
$$M_P = 8(\sqrt{P} - 1) - 3$$
$$M_P = 8\sqrt{P} - 8 - 3$$
$$M_P = 8\sqrt{P} - 11$$

Since $\sqrt{P}$ is just equal to the number of rows/columns (s) of the square Klotski, then the minimum number of moves (M) is

$$M = 8s - 11$$

Example 1

In a square Klotski with a 4×4 dimension (See Figure 2), by applying the aforementioned formula, the minimum number of moves is

$$M = 8(4) - 11$$
$$= 32 - 11$$
$$= 21$$

The result of example 1 conforms with the data for the minimum number of moves (See Figure 6).

Formula for the Minimum Number of Moves of a Rectangular Klotski;

$$(n+1)(n+k) \quad \{n|n \in \mathbb{Z}^+\} \quad \{k|k \in \mathbb{Z}^+, k > 1\}$$

The arithmetic sequence formula of the different minimum number of moves (M) for each $(n+1)(n+k)$

Minimum no. of moves for $(n+1)(n+2)$

$P_{(n+1)(n+2)}$	$M_{(n+1)(n+2)}$
6	9
12	17
20	25
30	33
...	...
$(n+1)(n+2)$	$8n+1$

Minimum no. of moves for $(n+1)(n+3)$

$P_{(n+1)(n+3)}$	$M_{(n+1)(n+3)}$
8	15
15	23
24	31
35	39
...	...
$(n+1)(n+3)$	$8n+7$

Minimum no. of moves for $(n+1)(n+4)$

$P_{(n+1)(n+4)}$	$M_{(n+1)(n+4)}$
10	21
18	29
28	37
40	45
...	...
$(n+1)(n+4)$	$8n+13$

Minimum no. of moves for $(n+1)(n+5)$

$P_{(n+1)(n+5)}$	$M_{(n+1)(n+5)}$
12	27
21	35
32	43
45	51
...	...
$(n+1)(n+5)$	$8n+19$

The following table summarizes the product and the minimum number of moves for $(n+1)(n+k)$ Klotski:

Product (P)	Minimum no. of moves (M)
$(n+1)(n+2)$	$8n+1$
$(n+1)(n+3)$	$8n+7$
$(n+1)(n+4)$	$8n+13$
$(n+1)(n+5)$	$8n+19$
$(n+1)(n+6)$	$8n+25$
$(n+1)(n+7)$	$8n+31$
...	...
$P=(n+1)(n+k)$	$M=8n+6k-11$

Formulation of $M = 8n+6k-11$

$$M = 8n + (1 + [k-2]6)$$
$$M = 8n + (1 + 6k - 12)$$
$$M = 8n + 1 + 6k - 12$$
$$M = 8n + 6k - 11$$

Derivation of n

$$P = (n+1)(n+k)$$
$$P = n^2 + kn + n + k$$
$$P = n^2 + (k+1)n + k$$
$$0 = n^2 + (k+1)n + k - P$$

using quadratic formula to find n

$$n = \frac{-(k+1) \pm \sqrt{(k+1)^2 - 4(1)(k-P)}}{2(1)}$$
$$n = \frac{-k-1 \pm \sqrt{k^2+2k+1-4k+4P}}{2}$$
$$n = \frac{-k-1 \pm \sqrt{k^2-2k+1+4P}}{2}$$
$$n = \frac{-k-1 \pm \sqrt{(k-1)^2+4P}}{2}$$

Minimum number of moves in terms of the product (M_P)

$$M = 8n + 6k - 11$$
$$M_P = 8\left(\frac{-k-1 \pm \sqrt{(k-1)^2+4P}}{2}\right) + 6k - 11$$
$$M_P = 4(-k - 1 \pm \sqrt{(k-1)^2 + 4P}) + 6k - 11$$
$$M_P = -4k - 4 \pm 4\sqrt{(k-1)^2 + 4P} + 6k - 11$$
$$M_P = 4\sqrt{(k-1)^2 + 4P} + 2k - 15$$

Since k is the difference (d) of the two sides $+1$,
$d = (n+k) - (n+1)$
$d = n + k - n - 1$
$d = k - 1$
$d + 1 = k$
$k = d + 1$

hence the minimum number of moves given the product and difference (M_{Pd}) is

$$M_{Pd} = 4\sqrt{((d+1)-1)^2 + 4P} + 2(d+1) - 15$$
$$M_{Pd} = 4\sqrt{(d+1-1)^2 + 4P} + 2d + 2 - 15$$
$$M_{Pd} = 4\sqrt{(d)^2 + 4P} + 2d - 13$$
$$M_{Pd} = 4\sqrt{d^2 + 4P} + 2d - 13$$

Furthermore, if we let l and w be the number of columns and rows and $l > w$, then the minimum number of moves of a rectangular Klotski (M) is

$$M_{Pd} = 4\sqrt{d^2 + 4P} + 2d - 13$$
$$M = 4\sqrt{(l-w)^2 + 4(l)(w)} + 2(l-w) - 13$$
$$M = 4\sqrt{l^2 - 2lw + w^2 + 4lw} + 2l - 2w - 13$$
$$M = 4\sqrt{l^2 + 2lw + w^2} + 2l - 2w - 13$$
$$M = 4\sqrt{(l+w)^2} + 2l - 2w - 13$$
$$M = 4(l+w) + 2l - 2w - 13$$
$$M = 4l + 4w + 2l - 2w - 13$$
$$M = 6l + 2w - 13$$

Example 2.1

In a rectangular Klotski whose product and difference of the dimensions are 21 and 4, respectively (See Figure 3), by applying the formula for the product and difference, the minimum number of moves is

$$\begin{aligned} M_{Pd} &= 4\sqrt{4^2 + 4(21)} + 2(4) - 13 \\ &= 4\sqrt{16 + 84} + 8 - 13 \\ &= 4\sqrt{100} - 5 \\ &= 4(10) - 5 \\ &= 40 - 5 \\ &= 35 \end{aligned}$$

Example 2.2

In a rectangular Klotski with a 7×11 dimension, by applying the aforementioned formula, the minimum number of moves is

$$\begin{aligned} M &= 6(11) + 2(7) - 13 \\ &= 66 + 14 - 13 \\ &= 67 \end{aligned}$$

The result of examples 2.1 & 2.2 also conforms with the data for the minimum number of moves (See Figure 6).

4 Chapter IV

4.1 Summary of the Study

To sum it up, the investigators found out that in the game Klotski puzzle, which aims to move one specific tile from the corner to its opposite corner, there is a technique where the player must move the tile diagonally in order to find the minimum number of moves. Varied formulae were derived and formulated to determine the minimum number of moves based on the dimensions of the Klotski puzzle. The investigators methodology for developing these formulae was based on the arithmetic sequence formed by the pattern whose common difference are 8 and 6.

4.2 Recommendation

The investigators recommend to study further not just on square and rectangles but also on triangle and other shaped Klotskis. The movement of more than 1 block or more than 1 free space can also be taken into consideration in the future conduct of related investigation. Also, blocks with diferent shapes and sizes will be a good topic for investigation. Lastly, the investigators of this study recommends future researchers to investigate the relevance of the results of this study to its other potential real life application/s.

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